HANNAH MORE
PRIMARY SCHOOL

Calculation Policy

Updated for the 2014 National Curriculum

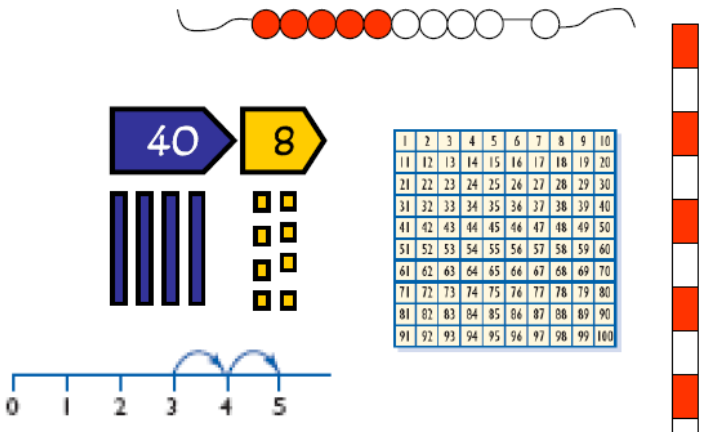
Progression in Teaching Addition

Mental Skills

Recognise the size and position of numbers
Count on in ones and tens
Know number bonds to 10 and 20
Add multiples of 10 to any number
Partition and recombine numbers
Bridge through 10

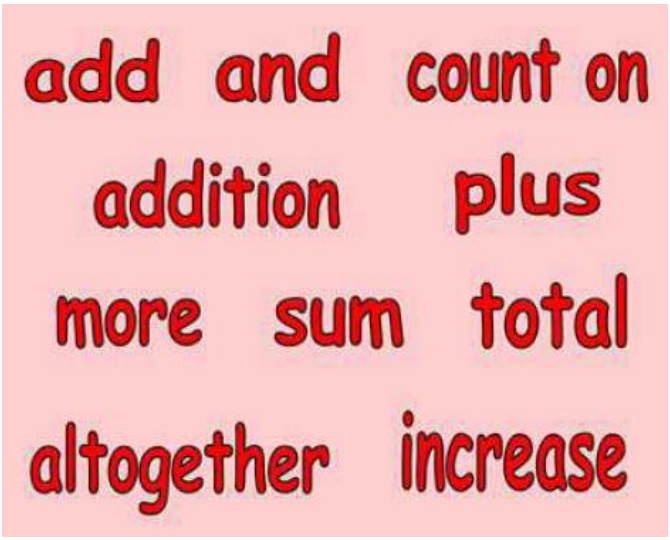
Models and Images

Counting apparatus
Place value apparatus
Place value cards
Number tracks
Numbered number lines
Marked but unnumbered number lines
Empty number lines
Hundred square
Counting stick
Bead string



Key Vocabulary

add
addition
plus
and
count on
more
sum
total
altogether
increase



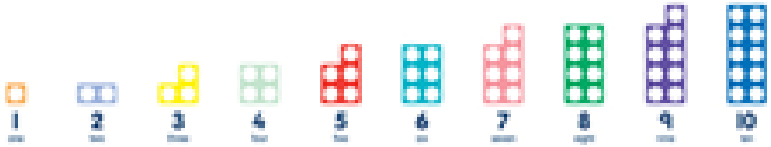
Reception

Recognise numbers 0 to 20

Count reliably up to 20 everyday objects

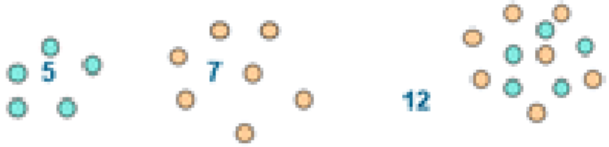
Numicon shapes are introduced straight away and can be used to:

- Identify 1 more/less
- Combine pieces to add
- Find number bonds
- Add without counting



Children can record this by printing or drawing around Numicon pieces.

Children begin to combine groups of objects using concrete apparatus.



Construct number sentences verbally or using cards to go with practical activities.

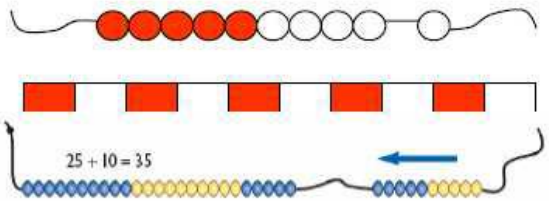
Children are encouraged to read number sentences aloud in different ways:
"Three add two equals 5" "5 is equal to three and two"

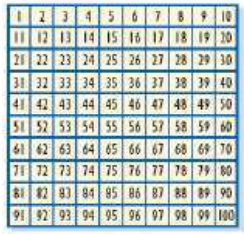
Children make a record in pictures, words or symbols of addition activities already carried out.

Number tracks can be introduce to count up on and to find one more:

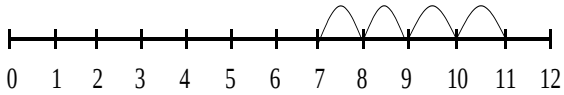
What is 1 more than 4? What is 2 less than 13?

Counts in ones and tens





Number lines can be used alongside number tracks, using practical apparatus such as bead strings to solve addition calculations and word problems.



7+ 4 = 12

Children will need opportunities to look at and talk about different models and images as they move between different representations.

Addition – Years 1 - 3

Year 1

+ = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

2 = 1+ 1

2 + 3 = 4 + 1

Missing numbers need to be placed in all possible places.

3 + 4 = □ □ = 3 + 4

3 + □ = 7 7 = □ + 4

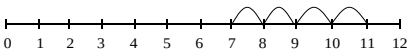
Counting and Combining sets of Objects

Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)

Understanding of counting on with a number track.

Understanding of counting on with a numberline (supported by models and images/apparatus such as bead strings).

7+ 4



Year 2

Missing number problems e.g 14 + 5 = 10 + □ 32 + □ + □ = 100

35 = 1 + □ + 5

It is valuable to use a range of representations. Children can use the bead string to support their number line work.

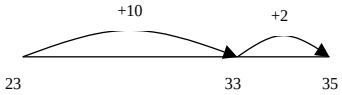
Continue to use number lines to develop understanding of:

Counting on in tens and ones

23 + 12 = 23 + 10 + 2

= 33 + 2

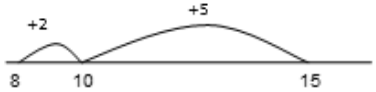
= 35



Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10 e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.

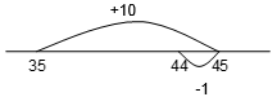
8 + 7 = 15



Adding 9 or 11 by adding 10 and adjusting by 1

e.g. Add 9 by adding 10 and adjusting by 1

35 + 9 = 44

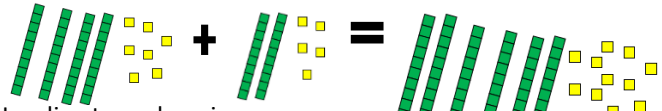


Towards a Written Method

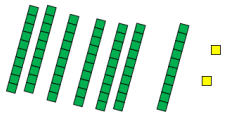
Partitioning in different ways and recombine

47+25

47 **25** **60 + 12**



Leading to exchanging:



72

Expanded written method

40 + 7 + 20 + 5 =

40+ 20 + 7 + 5 =

60 + 12 = 72

40 + 7
+ 20 + 5
60 + 12 = 72

Year 3

Missing number problems using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

Partition into tens and ones

Partition both numbers and recombine.

Count on by partitioning the second number only e.g.

247 + 125 = 247 + 100 + 20+ 5

= 347 + 20 + 5

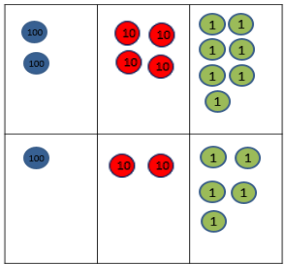
= 367 + 5

= 372

Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

Towards a Written Method

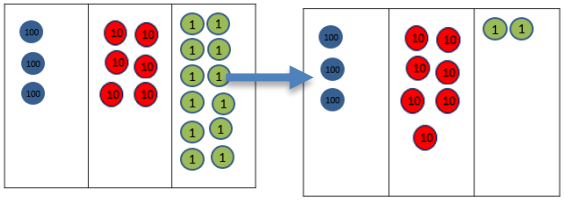
Introduce expanded column addition modelled with place value counters (Dienes could be used for those who need a less abstract representation)



200 + 40 + 7
100 + 20 + 5
300 + 60 +12 = 372

247
+125
12
60
300
372

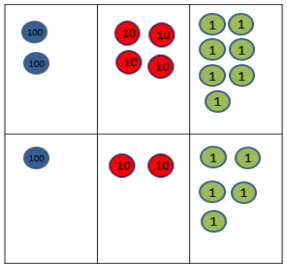
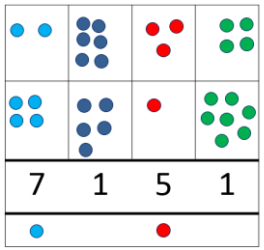
Leading to children understanding the exchange between tens and ones.



Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

247
+125
372
10

Addition – Years 4- 6

Year 4	Year 5	Year 6
Missing number/digit problems: **Mental methods** should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. **Written methods (progressing to 4-digits)** Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.  200 + 40 + 7 100 + 20 + 5 300 + 60 +12 = 372 247 +125 12 60 300 372 **Compact written method** Extend to numbers with at least four digits.  2634 +4517 7151 1 1 **Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.** Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits). 72.8 + 54.6 127.4 1 1	Missing number/digit problems: **Mental methods** should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency e.g. 12462 + 2300 = 14762 **Written methods (progressing to more than 4-digits)** As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm. 172.83 + 54.68 227.51 1 1 1 Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.	Missing number/digit problems: **Mental methods** should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. **Written methods** As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. Continue calculating with decimals, including those with different numbers of decimal places **Problem Solving** Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding.

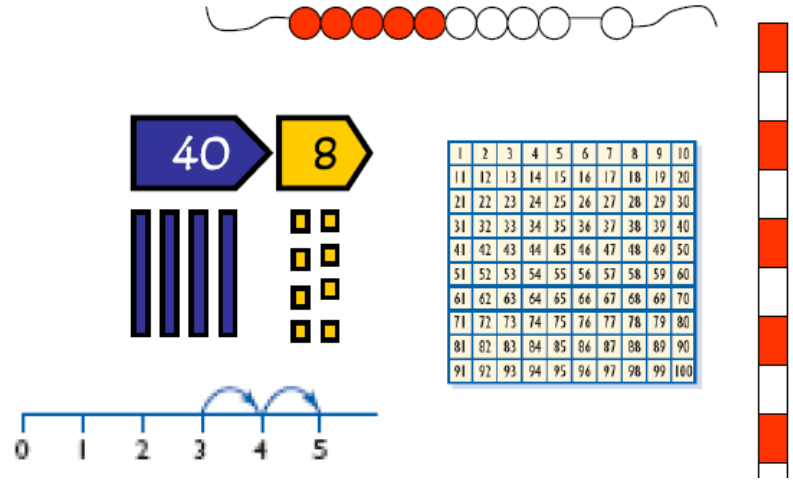
Progression in Teaching Subtraction

Mental Skills

Recognise the size and position of numbers
Count back in ones and tens
Know number facts for all numbers to 20
Subtract multiples of 10 from any number
Partition and recombine numbers (only partition the number to be subtracted)
Bridge through 10

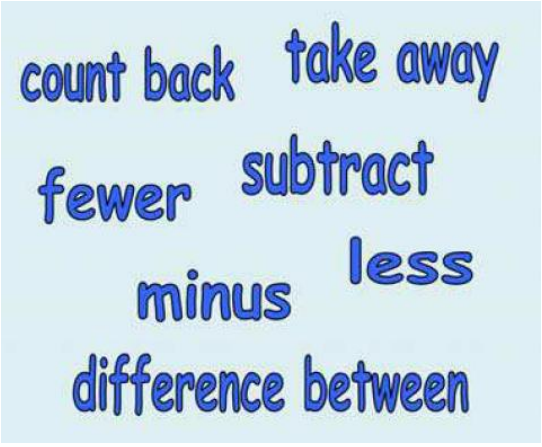
Counting apparatus

Place value apparatus
Place value cards
Number tracks
Numbered number lines
Marked but unnumbered lines
Hundred square
Empty number lines.
Counting stick
Bead strings



Vocabulary

subtract
take away
minus
count back
less
fewer
difference between

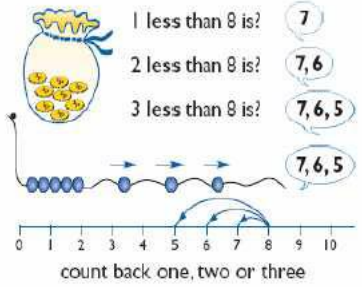


Reception

Begin to count backwards in familiar contexts such as number rhymes or stories.

Children begin with mostly pictorial representations.

Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.



Construct number sentences verbally or using cards to go with practical activities.

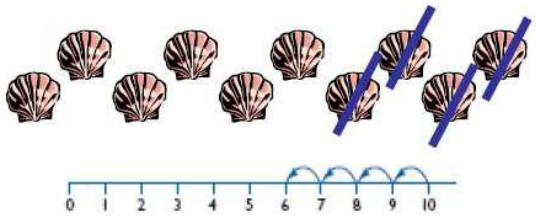
Children are encouraged to read number sentences aloud in different ways “five subtract one leaves four” “four is equal to five subtract one”

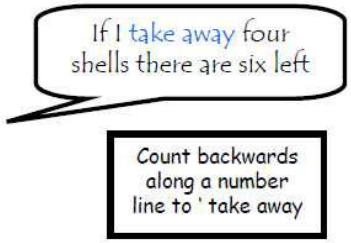
Children make a record in pictures, words or symbols of subtraction activities already carried out.

Number tracks can be introduced to find one less.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Number lines can be used alongside number tracks, practical apparatus (such as bead strings) to solve subtraction calculations and word problems.
Children count back under a number line.





Children will need opportunities to look and talk about different models and images as they move between representations.

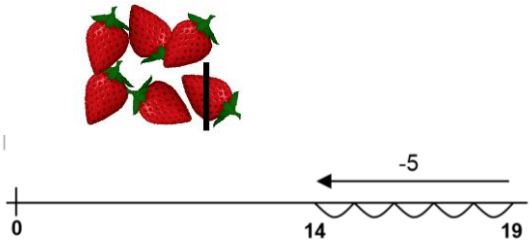
Subtraction – Year 1- 3

Year 1

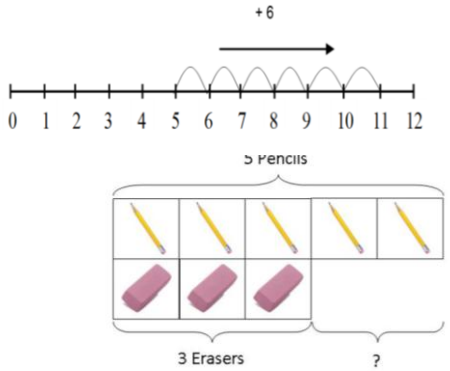
Missing number problems e.g. 7 = □ - 9; 20 - □ = 9; 15 – 9 = □; □ - □ = 11; 16 – 0 = □

Use concrete objects and pictorial representations. If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown.

Understand subtraction as take-away:



Understand subtraction as finding the difference:



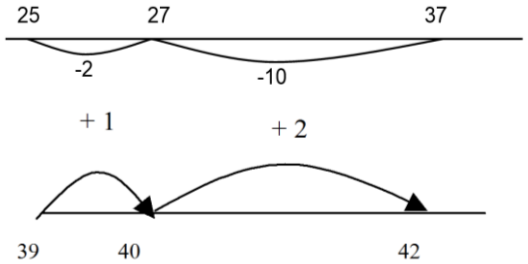
The above model **(the bar model)** would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation.

The use of other images is also valuable for modelling subtraction e.g. Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings

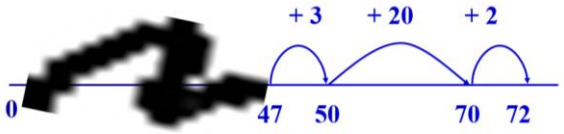
Year 2

Missing number problems
e.g. 52 – 8 = □; □ – 20 = 25; 22 = □ – 21; 6 + □ + 3 = 11

It is valuable to use a range of representations (also see Y1). Continue to use number lines to model take-away and difference.
E.g:



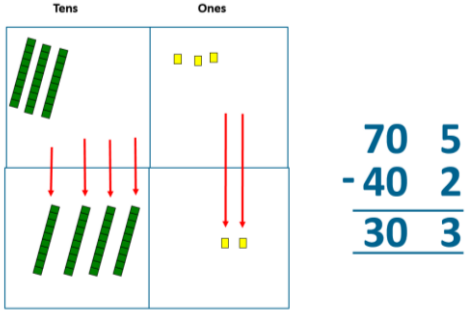
The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.



The bar model should continue to be used, as well as images in the context of **measures**.

Towards written methods

Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus. E.g. 75 – 42



$$\begin{array}{r} 70\ \ 5 \\ -40\ \ 2 \\ \hline 30\ \ 3 \end{array}$$

Year 3

Missing number problems e.g. □ = 43 – 27; 145 – □ = 138; 274 – 30 = □; 245 – □ = 195; 532 – 200 = □; 364 – 153 = □

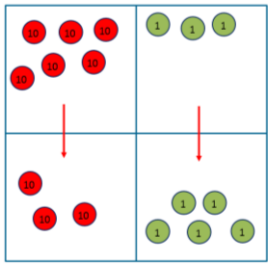
Mental methods should continue to develop, supported by a range of models and images, including the number line.

The bar model should continue to be used to help with problem solving (see Y1 and Y2).

Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.

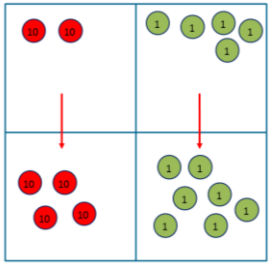
Written methods (progressing to 3-digits)

Introduce expanded column subtraction with no decomposition, modelled with place value counters (Dienes could be used for those who need a less abstract representation)



$$\begin{array}{r} 90\ \ 8 \\ -30\ \ 5 \\ \hline 60\ \ 3 \end{array}$$

For some children this will lead to exchanging, modelled using place value counters (or Dienes).

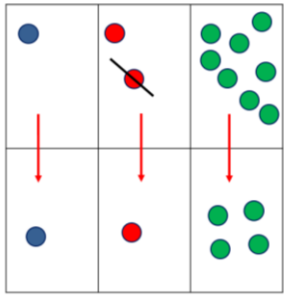
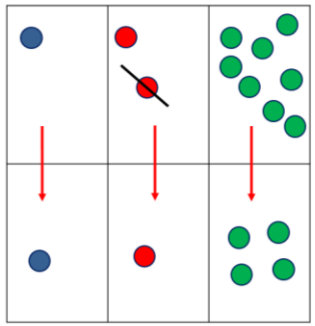
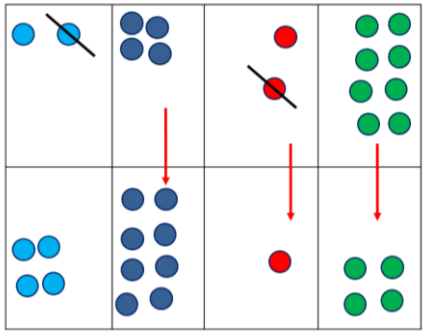


$$\begin{array}{r} {}^{60}\cancel{70}\ {}^{1}2 \\ -40\ \ 7 \\ \hline 20\ \ 5 \end{array}$$

A number line and expanded column method may be compared next to each other.

Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

Subtraction – Year 4 – 6

Year 4	Year 5	Year 6
Missing number/digit problems: 456 + □ = 710; 1□7 + 6□ = 200; 60 + 99 + □ = 340; 200 – 90 – 80 = □; 225 - □ = 150; □ – 25 = 67; 3450 – 1000 = □; □ - 2000 = 900 **Mental methods** should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. **Written methods (progressing to 4-digits)** Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.  200 20 12 - 100 10 4 100 10 8 If understanding of the expanded method is secure, children will move on to the formal method of decomposition, which again can be initially modelled with place value counters.  2 1 232 - 114 118	**Missing number/digit problems:** 6.45 = 6 + 0.4 + □; 119 - □ = 86; 1 000 000 - □ = 999 000; 600 000 + □ + 1000 = 671 000; 12 462 – 2 300 = □ **Mental methods** should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. **Written methods (progressing to more than 4-digits)** When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters.  5 1 2 1 6232 - 4814 1418 Progress to calculating with decimals, including those with different numbers of decimal places.	**Missing number/digit problems:** □ and # each stand for a different number. # = 34. # + # = □ + □ + #. What is the value of □? What if # = 28? What if # = 21 10 000 000 = 9 000 100 + □ 7 – 2 x 3 = □; (7 – 2) x 3 = □; (□ - 2) x 3 = 15 **Mental methods** should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. **Written methods** As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with decomposition to be secured. Teachers may also choose to introduce children to other efficient written layouts which help develop conceptual understanding. For example: 326 -148 -2 -20 200 178 Continue calculating with decimals, including those with different numbers of decimal places.

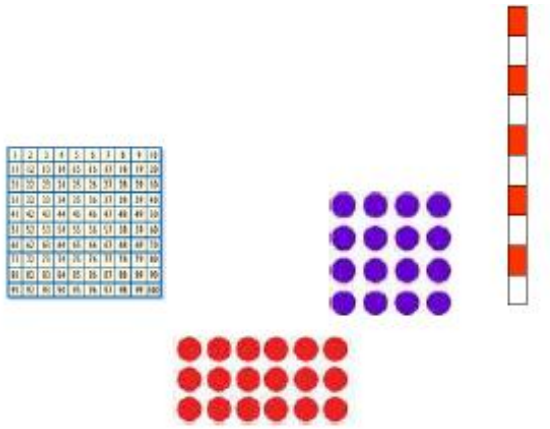
Progression in Teaching Multiplication

Mental Skills

Recognise the size and position of numbers
Count on in different steps 2s, 5s, 10s
Double numbers up to 10
Recognise multiplication as repeated addition
Quick recall of multiplication facts
Use known facts to derive associated division facts
Use known facts to generate other facts (e3.g. double the 2 x table to find 4 x table)
Multiplying by 10, 100, 1000 and understanding the effect

Counting apparatus

Place value apparatus
Arrays
100 squares
Number tracks
Numbered number lines
Marked but unnumbered lines
Empty number lines
Multiplication squares
Counting stick
Bead strings



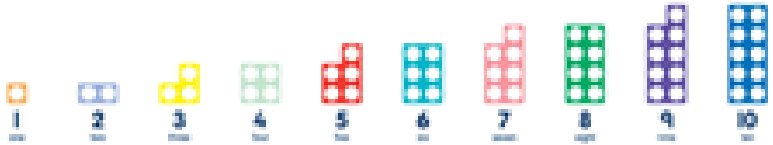
Vocabulary

lots of
groups of
times
multiply
multiplication
multiple
product
once, twice, three times
array, row, column
double
repeated addition

Multiplication - Reception

The link between addition and multiplication can be introduced through doubling.

Numicon is used to visualise the repeated adding of the same number. These can be drawn around or printed as a way of recording.



Children begin with mostly pictorial representations.

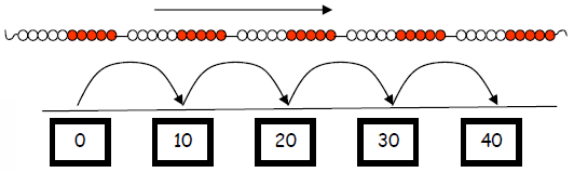
How many groups of 2 are there?

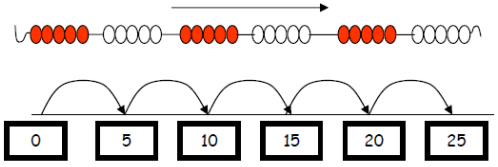
Real life contexts and use of practical equipment to count in repeated groups of the same size.

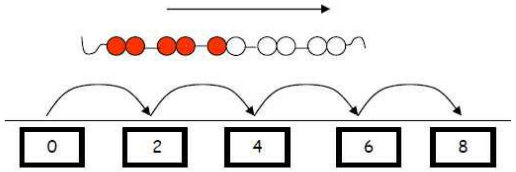
How many wheels are there altogether?

How much money do I have?

Count in twos; fives; tens both aloud and with objects







Children are given multiplication problems set in real life context. Children are encouraged to visualise the problem.

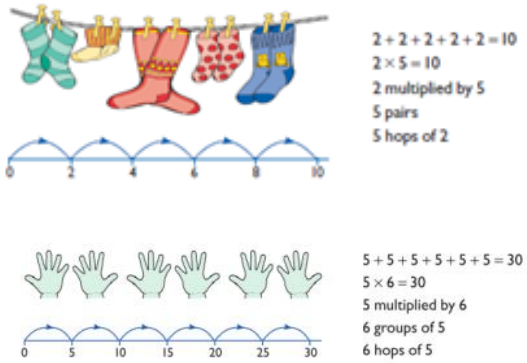
Children are encouraged to read number sentences aloud in different ways “five times two makes ten” “ten is equal to five multiplied by two”

Multiplication – Year 1 - 3

Year 1

Understand multiplication is related to doubling and combing groups of the same size (repeated addition)

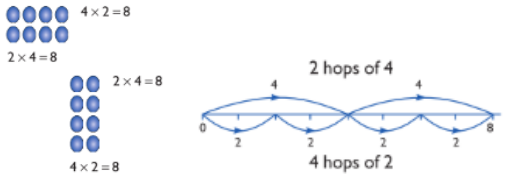
Washing line, and other practical resources for counting. Concrete objects. Numicon; bundles of straws, bead strings



Problem solving with concrete objects (including money and measures

Use cuissenaire and bar method to develop the vocabulary relating to 'times' –
Pick up five, 4 times

Use arrays to understand multiplication can be done in any order (commutative)

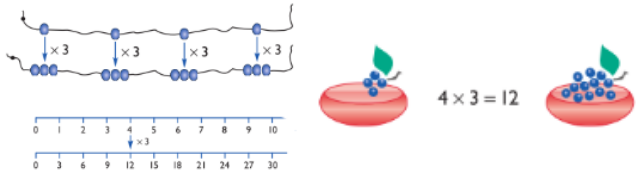


Year 2

Expressing multiplication as a number sentence using x
Using understanding of the inverse and practical resources to solve missing number problems.

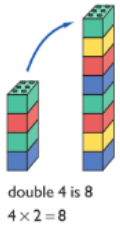
7 x 2 = □	□ = 2 x 7
7 x □ = 14	14 = □ x 7
□ x 2 = 14	14 = 2 x □
□ x ◯ = 14	14 = □ x ◯

Develop understanding of multiplication using array and number lines (see Year 1).
Include multiplications not in the 2, 5 or 10 times tables.

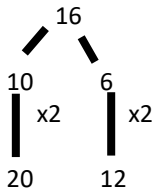


Begin to develop understanding of multiplication as scaling (3 times bigger/taller)

Doubling numbers up to 10 + 10
Link with understanding scaling
Using known doubles to work out double 2d numbers
(double 15 = double 10 + double 5)



Towards written methods
Use jottings to develop an understanding of doubling two digit numbers.



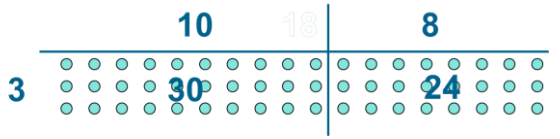
Children should be able to recall the x2 , x5 and x10 times tables.

Year 3

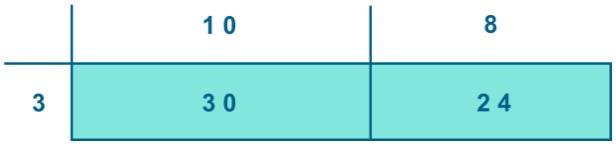
Missing number problems
Continue with a range of equations as in Year 2 but with appropriate numbers.

Mental methods
Doubling 2 digit numbers using partitioning
Demonstrating multiplication on a number line – jumping in larger groups of amounts
13 x 4 = 10 groups 4 = 3 groups of 4

Written methods (progressing to 2d x 1d)
Developing written methods using understanding of visual images



Develop onto the grid method



Give children opportunities for children to explore this and deepen understanding using Dienes apparatus and place value counters

Children should be able to recall the x3 , x4 and x8 times tables as well as the times tables learned before in Year 2.

Multiplication – Year 4-6

Year 4

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

□2 x 5 = 160

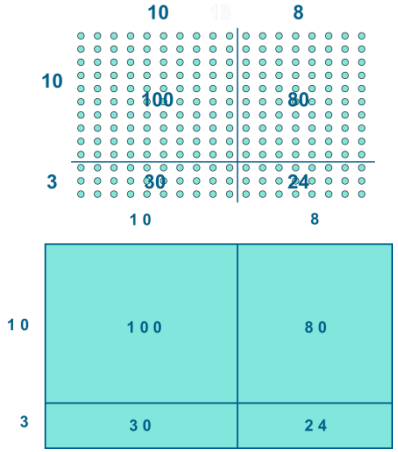
Mental methods

Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25cm sunflower be if it grew 6 times taller?)

Written methods (progressing to 3d x 2d)

Children to embed and deepen their understanding of the grid method to multiply up 2d x 2d. Ensure this is still linked back to their understanding of arrays and place value counters.



Then move onto:

30	+ 8	
X	7	
210		30 x 7 = 210
56		8 x 7 = 56
266		

Children should be able to recall all times tables up to x12.

Year 5

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits.

Mental methods

X by 10, 100, 1000 using moving digits ITP

Use practical resources and jottings to explore equivalent statements (e.g. 4 x 35 = 2 x 2 x 35)

Recall of prime numbers up 19 and identify prime numbers up to 100 (with reasoning)

Solving practical problems where children need to scale up.

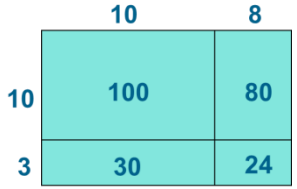
Relate to known number facts.

Identify factor pairs for numbers

Written methods (progressing to 4d x 2d)

Long multiplication using place value counters

Children to explore how the grid method supports an understanding of long multiplication (for 2d x 2d)



Become confident with:

30	+ 8	
X	7	
210		30 x 7 = 210
56		8 x 7 = 56
266		

Once confident, move onto:

38	
X	7
210	
56	
266	

Year 6

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

Mental methods

Identifying common factors and multiples of given numbers

Solving practical problems where children need to scale up.

Relate to known number facts.

Written methods

Continue to refine and deepen understanding of written methods including fluency for using long multiplication

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

Once confident, move onto:

124 × 26 becomes

	1	2		
	1	2	4	
×		2	6	
	7	4	4	
2	4	8	0	
3	2	2	4	
1	1			

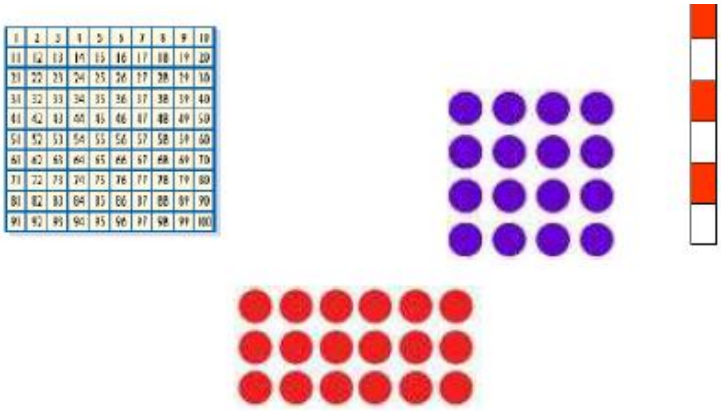
Answer: 3224

Children need to then move through to work with decimals, and larger numbers as per the curriculum document.

Progression in Teaching Division

Mental Skills
Recognise the size and position of numbers
Count back in different steps 2s, 5s, 10s
Halve numbers to 20
Recognise division as repeated subtraction
Quick recall of division facts
Use known facts to derive associated facts
Divide by 10, 100, 1000 and understanding the effect
Divide by multiples of 10

Counting apparatus
Arrays
100 squares
Number tracks
Numbered number lines
Marked but unnumbered lines
Empty number lines
Multiplication squares



Vocabulary
lots of
groups of
share
group
halve
half
divide
division
divided by
remainder
factor
quotient
divisible

Division and Fractions - Reception

Division:

Children need to solve problems including doubling, halving and sharing.

Children need to see and hear representations of division as both grouping and sharing.

Grouping model

Mum has 6 socks. She grouped them into pairs – how many pairs did she make?

Sharing model

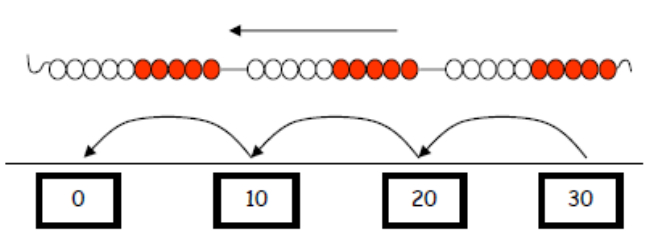
I have 10 sweets. I want to share them with my friend. How many will we have each?

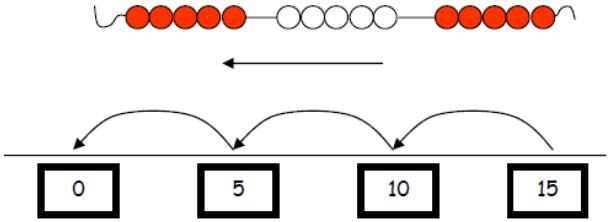
Division can be introduced through halving.

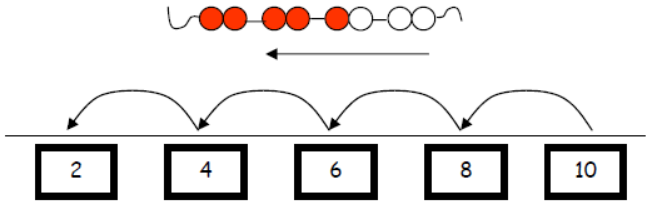
Children begin with mostly pictorial representations linked to real life contexts.

Children begin to record the calculation that has been carried out.

Children begin to count back in twos, fives and tens.







Fractions:

Set the problems in real life context and solve them with concrete apparatus that will support children's understanding.

"I have got 5 bones to share between my two dogs. How many bones will they get each?"

Children have a go at recording the calculation that has been carried out.

2 ½ + 2 ½ = 5

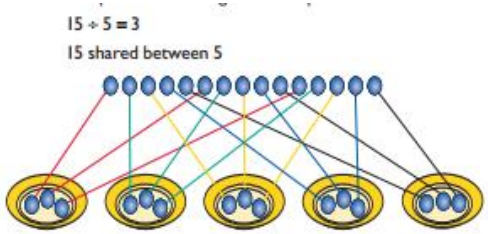
Division – Year 1-3

Year 1

Children must have secure counting skills- being able to confidently count in 2s, 5s and 10s.

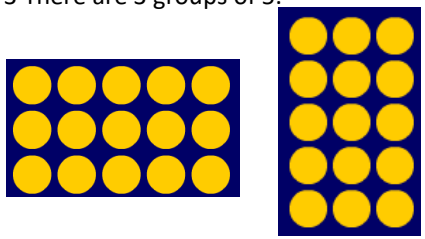
Children should be given opportunities to reason about what they notice in number patterns.

Group AND share small quantities- understanding the difference between the two concepts.
Sharing - Develops importance of one-to-one correspondence.



Children should be taught to share using concrete apparatus.
Grouping - Children should apply their counting skills to develop some understanding of grouping.

Use of arrays as a pictorial representation for division.
15 ÷ 3 = 5 There are 5 groups of 3.
15 ÷ 5 = 3 There are 3 groups of 5.



Children should be able to find ½ and ¼ and simple fractions of objects, numbers and quantities.

Year 2

÷ = signs and missing numbers

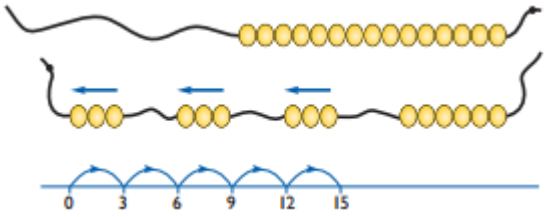
6 ÷ 2 = □ □ = 6 ÷ 2
6 ÷ □ = 3 3 = 6 ÷ □
□ ÷ 2 = 3 3 = □ ÷ 2
□ ÷ ∇ = 3 3 = □ ÷ ∇

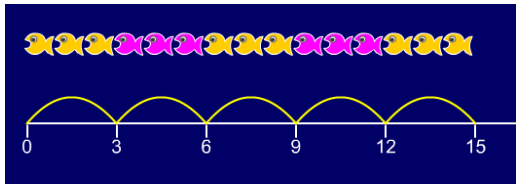
Know and understand sharing and grouping- introducing children to the ÷ sign.

Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

Grouping using a numberline
Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'.

15 ÷ 3 = 5



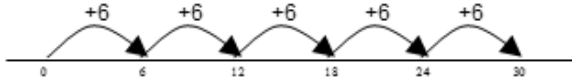


Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see?

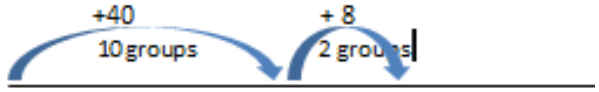
Year 3

÷ = signs and missing numbers
Continue using a range of equations as in year 2 but with appropriate numbers.

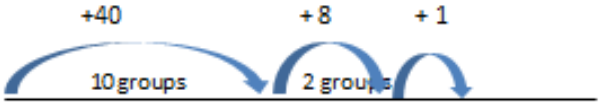
Grouping
How many 6's are in 30?
30 ÷ 6 can be modelled as:



Becoming more efficient using a numberline
Children need to be able to partition the dividend in different ways.
48 ÷ 4 = 12



Remainders
49 ÷ 4 = 12 r1



Sharing – 49 shared between 4. How many left over?
Grouping – How many 4s make 49. How many are left over?

Place value counters can be used to support children apply their knowledge of grouping.
For example:
60 ÷ 10 = How many groups of 10 in 60?
600 ÷ 100 = How many groups of 100 in 600?

Division – Year 4-6

Year 4

÷ = signs and missing numbers

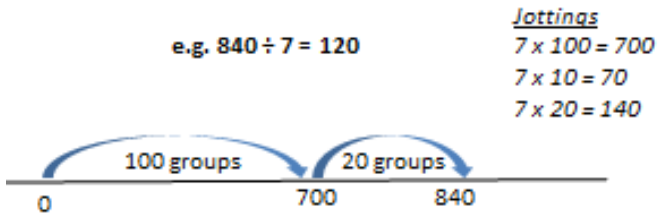
Continue using a range of equations as in year 3 but with appropriate numbers.

Sharing, Grouping and using a number line

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding.

Children should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:
- Dividend just over 10x the divisor, e.g. 84 ÷ 7
- Dividend just over 10x the divisor when the divisor is a teen number, e.g. 173 ÷ 15 (lea rning sensible strategies for calculations such as 102 ÷ 17)
- Dividend over 100x the divisor, e.g. 840 ÷ 7
- Dividend over 20x the divisor, e.g. 168 ÷ 7



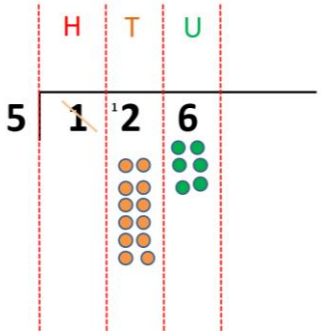
All of the above stages should include calculations with remainders as well as without.

Remainders should be interpreted according to the context. (i.e. rounded up or down to relate to the answer to the problem)

Formal Written Methods

Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)

Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends. E.g. fig 1



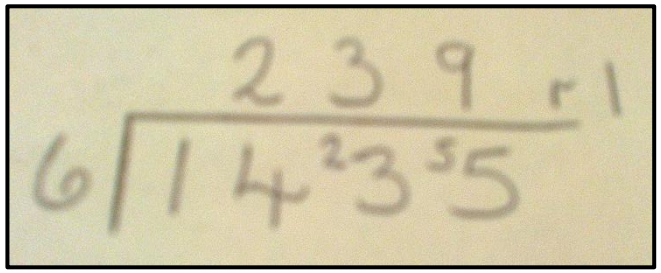
Year 5

Formal Written Methods

Continued as shown in Year 4, leading to the efficient use of a formal method.

The language of grouping to be used (see link from fig. 1 in Year 4)

E.g. 1435 ÷ 6



Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction.

Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)

Year 6

÷ = signs and missing numbers

Continue using a range of equations but with appropriate numbers

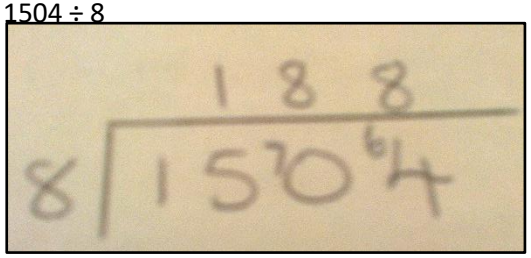
Sharing and Grouping and using a number line

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate.

Quotients should be expressed as decimals and fractions

Formal Written Methods – long and short division

E.g. 1504 ÷ 8



E.g. 2364 ÷ 15